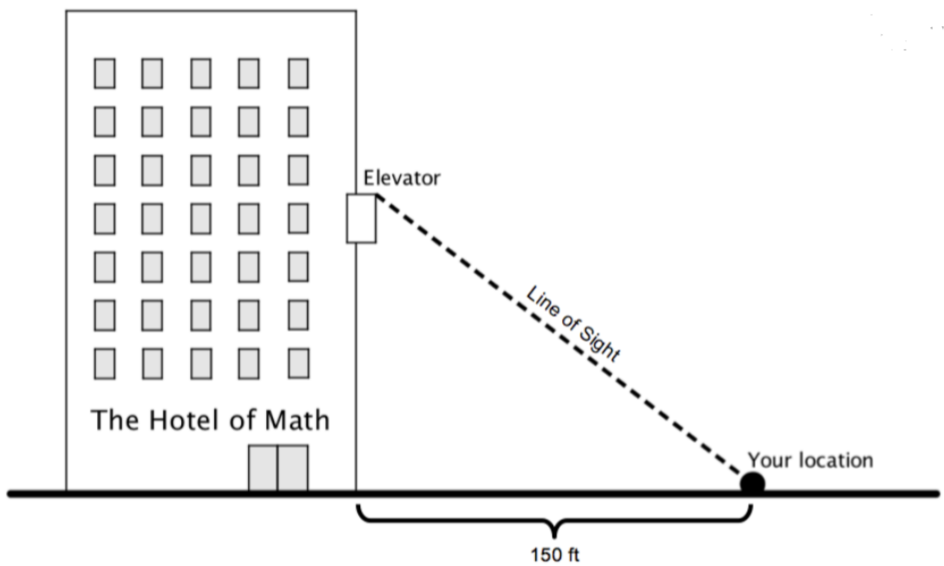
Related Rates

1. While sitting at an outdoor restaurant in a large city you notice that a hotel has an elevator on the outside of the building. You are 150 feet away from the hotel.



What is the relationship between the rate of change of the height, h, of the elevator and the rate of change of the angle, θ, between the ground and your line of sight?

(a) $\dfrac{d\theta}{dt} = \left(h^2 + 150^2\right)\dfrac{dh}{dt}$

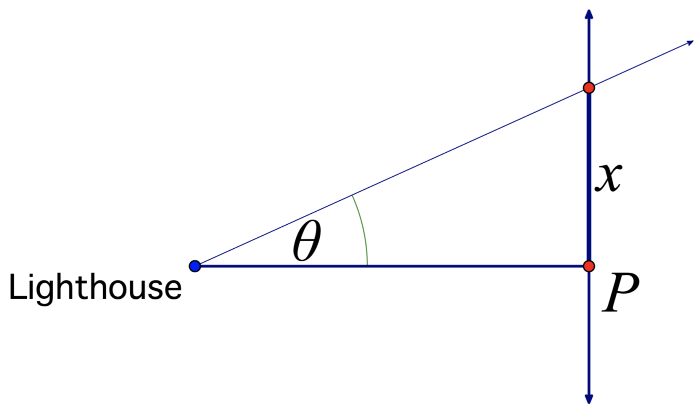
(b) $\tan\dfrac{d\theta}{dt} = \dfrac{1}{150}\dfrac{dy}{dt}$

(c) $\sec^2\theta\,\dfrac{d\theta}{dt} = \dfrac{1}{150}\dfrac{dy}{dt}$

(d) $\dfrac{d\theta}{dt} = \sqrt{150^2 - 2h}\dfrac{dh}{dt}$

(e) $\dfrac{d\theta}{dt} = 75h\dfrac{dh}{dt}$

2. A lighthouse is located on a small island 3 kilometers away from the nearest point P on a straight shoreline. Let x represent the distance between P and the light beam's intersection with the shoreline. Also let θ represent the measure of the angle created by the beam of light and the line connecting the lighthouse and P. Which formula defines the relationship between the rate of change of the angle's measure and the rate at which the beam of light is moving along the shoreline?



(a) $\tan\left(\frac{d\theta}{dt}\right) = \frac{dx}{dt}$

(b) $\sec^2(\theta) \cdot \frac{d\theta}{dt} = \frac{1}{3} \cdot \frac{dx}{dt}$

(c) $\sec^2\left(\frac{d\theta}{dt}\right) = \frac{1}{3} \cdot \frac{dx}{dt}$

(d) $\frac{d\theta}{dt} = \frac{1}{3\sec^2(\theta)}$

(e) $\frac{d\theta}{dt} = \frac{1}{1 + \left(\frac{dx}{dt}\right)^2}$

3. A cylindrical container of fixed radius r is being filled with water. Which of the following equations expresses the relationship between the rate of change of the volume V of the water in the container (with respect to time) and the rate of change of the height h of the water in the container (with respect to time)?

(a) $\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt}$

(b) $V = \pi r^2 h$

(c) $\frac{dV}{dt} = 2\pi r h + \pi r^2 \cdot \frac{dh}{dt}$

(d) $V = 2\pi r h$

(e) $\frac{dh}{dt} = \pi r^2 h$

4. A spherical ice ball of radius r is melting in a liquid. It melts in a uniform fashion so that it remains a sphere while melting. Which of the following equations expresses the relationship between the rate of change of the volume V of the ice (with respect to time) and the rate of change of its radius r (with respect to time)?

   (a) $\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

   (b) $\frac{dV}{dt} = 4\pi r^2$

   (c) $V = \frac{4}{3}\pi r^3$

   (d) $\frac{dV}{dt} = 4\pi \left(\frac{dr}{dt}\right)^2$

   (e) $\frac{dV}{dt} = \frac{4}{3}\pi r^3 \cdot \frac{dr}{dt}$

5. An ice cube that is initially three inches wide is placed on a table and starts to melt. (Assume that the ice cube melts in a uniform fashion so that at every instant it remains a cube.) Let V denote the volume of the cube, measured in cubic inches, let t denote the number of minutes elapsed since the cube began to melt, and let x denote the width of the cube, measured in inches. If we know that the values of x are related to the values of t according to the formula $x = 3e^{-t}$, then which of the following formulas correctly gives the instantaneous rate of change in V with respect to t?

   (a) $V'(t) = -81e^{-3t}$

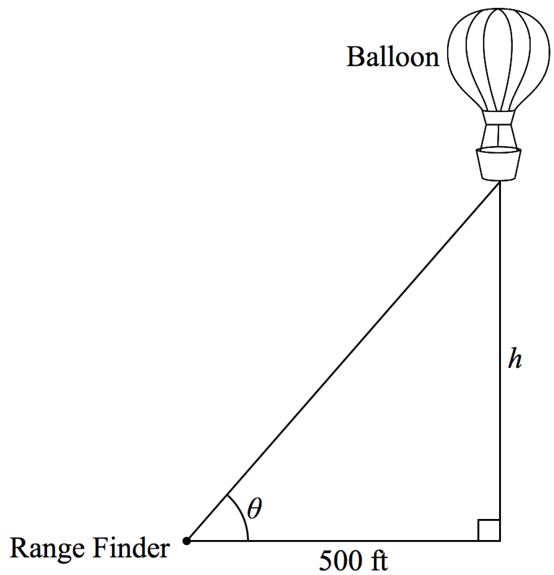
   (b) $V'(t) = \frac{27e^{-3t} - 27}{t}$

   (c) $V'(t) = 3e^{-t}$

   (d) $V'(t) = -3e^{-t}$

   (e) $V'(t) = 9e^{-3t}$

6. A hot air balloon rising straight up from a level field is tracked by a range finder 500 feet from the lift-off point (see the image below). At the moment the range finder's elevation angle is $\pi/4$ radians, the angle is increasing at a rate of 0.14 radians per minute. How fast is the balloon rising at that moment?



7. (*10 points*) Suppose gravel is being poured into a conical pile at a rate of 5 m^3/s, and suppose that the radius r of this cone is always half its height h. How fast is the height of the pile increasing when the height is 10 m?

(*Note that the formula for the volume of a cone is* $V = \frac{1}{3}\pi r^2 h$).